Sparse grid surrogate models for electromagnetic problems with many parameters

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A sparse grid surrogate model (or metamodel) is proposed to reduce the time-consumption involved by precise electromagnetic simulators. Though sparse grids have already been used in many other domains, such electromagnetic application appears to be original. The method can treat a high number of independent parameters that are intractable for many other techniques due to the "curse of dimensionality". The capabilities are illustrated via an example drawn from electromagnetic nondestructive evaluation.

***Index Terms*—nondestructive testing, interpolation, database, metamodeling, sparse grid**

I. Introduction

In many applications of computational electromagnetics, like inverse problems in nondestructive evaluation or design optimisation, the numerical simulation of the phenomenon has to be repeated many times with different combinations of the input parameters, making the complete process very time-consuming. To reduce the computational burden, the true simulator can be replaced by a cheap-to-evaluate surrogate model (or metamodel), consisting in an interpolant fitted to some pre-calculated samples of the true simulation. The key to provide an accurate interpolant is to choose the samples that span the approximation space in a certain-sense optimal manner. Just to name a few contributions, classical methods (e.g., variants of Latin Hypercube Sampling) are discussed in general in [1], whereas some recent results on adaptive sampling to electromagnetic problems are found, e.g., in [2]. However, these sampling methods are typically limited to cases where the number of input parameters does not exceed circa 6, due to the "curse-of-dimensionality". This limitation can be overcome by the sparse grid technique as proposed herein. Similar approaches have recently been applied to macroeconomic problems with success: models up to 24 input parameters are resolved in [3].

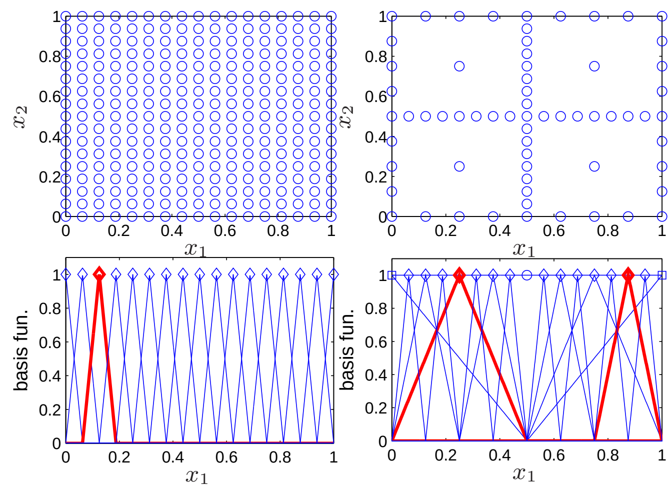

Fig. 1. Illustration of a full (left) and a sparse (right) grid in $N = 2$ dimensions. *Top*: samples. *Bottom*: basis functions along one axis (Some are highlighted for a better visualisation). The 2D basis functions are the tensor product of 1D basis functions defined in each dimension. Both grids use 17 equispaced samples (and so basis functions) per dimension, however, in the sparse grid, many of the nodes are suppressed. Sample numbers are 289 and 65, respectively.

II. Sparse grid interpolation as surrogate model

Let the vector $\mathbf{p}$ contain the N input parameters of the numerical simulation, and let $\mathbf{q}$ be the vector (length M) of the simulation results (outputs). For instance, in a nondestructive test, $\mathbf{p}$ describes the defect geometry and $\mathbf{q}$ consists in the samples of the observable signal. The simulation is referred to as an operator: $\mathbf{q} = \mathcal{F}\{\mathbf{p}\}$.

Let all input parameters be scaled to the $[0, 1]$ interval, i.e., the input vectors live in the N dimensional unit-hypercube $\mathbb{P} \in [0, 1]^N$. The sample set $(\mathbf{p}_i, \mathbf{q}_i)$, $i = 1, 2, \ldots, n$, that will be used to support the interpolant, can be generated by several strategies. The classical full grid approach places K equispaced samples along each coordinate of $\mathbb{P}$ to span a grid of totally $n = K^N$ samples. A piecewise N-linear interpolation can then be defined between the grid points. On the contrary, the sparse grid technique operates with a hierarchical set of linear basis functions as interpolants, and only a small portion of the nodes in a full grid is needed to establish the interpolation [4]. In Fig. 1, illustrations of the sample pattern are shown. For sufficiently smooth functions (with bounded mixed derivatives), the loss of interpolation accuracy is quite small compared to the gain in the reduction of sample number when changing from full to sparse grids. A detailed presentation will be given in the full paper.

The interpolation of $\mathcal{F}$ (having a vector output) can easily be traced back to M scalar interpolation tasks with a small increase of the computational cost.

III. The test problem: magnetic flux leakage

The test problem is drawn from the magnetic flux leakage (MFL) nondestructive testing method, that is used to detect and characterise surface degradations of ferromagnetic specimens.

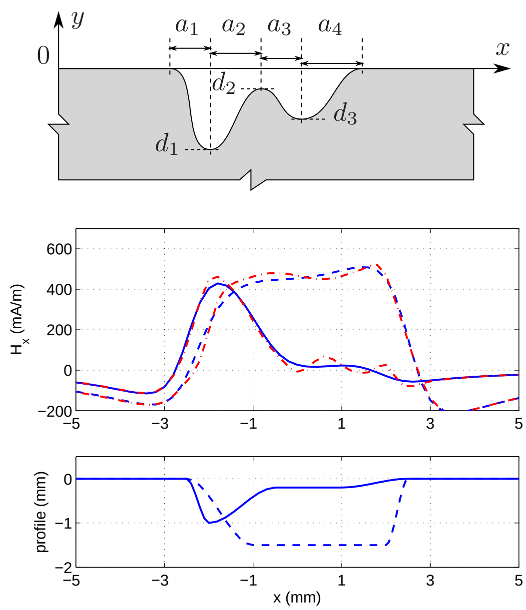

Fig. 2. *Top:* profile of the flaw and its 7 parameters. The four segments are 3^{rd} order polynomials. $a_1 \dots a_4$ vary between $[0.5 \dots 1.5]$mm; $d_1 \dots d_3$ vary between $[0.2 \dots 1.5]$mm, respectively. *Bottom:* Examples for groove profiles with corresponding signals along the scan line. The dash-dotted line is the interpolant based on a sparse grid database.

In our fictitious case (Fig. 2), a planar surface ($y = 0$) separates the ferromagnetic half-space ($\mu_r = 100$) and the air in the flawless case. An x-directed, homogeneous magnetic field is incident (e.g., by a permanent magnet yoke): $\mathbf{H}_0 = \hat{\mathbf{e}}_x(1\,\text{A/m})$. The surface is corrupted by grooves (long in the z direction and characterised by the profile in the xy plane). The distortion of the x-component of the magnetic field ($H_x(x)$) due to the groove is recorded above the damaged zone at a height of $0.5\,\text{mm}$ along a line of $10\,\text{mm}$ in the x direction, in 51 equidistant steps, resulting in an output vector $\mathbf{q}$ of length $M = 51$. The profile is described by four segments of cubic polynomials, having totally $N = 7$ parameters, i.e., $\mathbf{p} = [a_1, a_2, a_3, a_4, d_1, d_2, d_3]$.

The variation of the magnetic field is numerically calculated by introducing magnetic surface charges and by solving the arising integral equation via the Method of Moments [5]. Examples of the groove profile and the corresponding (computed) $H_x(x)$ signals are given in Fig. 2.

IV. Numerical results

The quality of interpolation is characterised by the discrepancy between the true signal $\mathbf{q}$ and the interpolated signal $\tilde{\mathbf{q}}$ as $\varepsilon(\mathbf{p}) = \|\tilde{\mathbf{q}} - \mathbf{q}\|$, with $\|\cdot\|$ being the Euclidean vector norm. A random set of 100 test points $\mathbf{p}_1, \dots, \mathbf{p}_{100}$ is chosen where ε is evaluated. The maximum ($\varepsilon_{\max}$) and the root mean square (ε_{RMS}) values will be given herein.

The sparse grid sampling and the interpolation is carried out by using the `spinterp` Matlab Toolbox [6]. The interpolation accuracy is compared to full grids with different levels of refinement in Fig. 3. Let us notice that the sparse grid can provide an accuracy in terms of $\varepsilon_{\max}$ around $\varepsilon_{\max,\text{best}} = 2.6\,\text{mA/m}$ using approximately 10 times less samples than

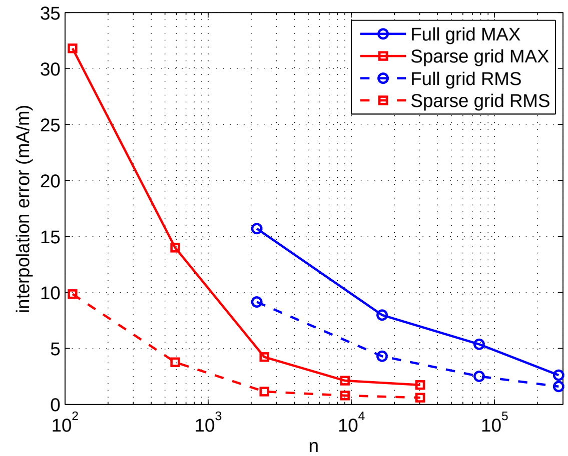

Fig. 3. Interpolation error with respect to the sample number, using sparse and full grid samplings.

the full grid. To give an insight to the relative error, let us note that the largest and the smallest norms of output signals occurring in the database are $(\|\mathbf{q}\|)_{\max} = 346\,\text{mA/m}$ ($\varepsilon_{\max,\text{best}}/(\|\mathbf{q}\|)_{\max} = 0.75\,\%$) and $(\|\mathbf{q}\|)_{\min} = 38.5\,\text{mA/m}$ ($\varepsilon_{\max,\text{best}}/(\|\mathbf{q}\|)_{\min} = 6.8\,\%$), respectively.

V. Conclusion, perspectives

The sparse grid interpolation is found to be a powerful tool for the interpolation of the input-output operator in a MFL-testing case with 7 defect parameters. This surrogate model is expected to be of great use in the fast solution of the corresponding inverse problem. In the full paper, more examples –involving more defect parameters– will be given. The adaptive generation of the sparse grid will also be addressed.

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